DIOPHANTINE ANALYSIS.

120. Proposed by L. E. DICKSON, Ph. D., The University of Chicago.

Find the prime numbers p for which $x^2-pxz-px-z+p^2-3=0$ has more than two sets of positive integral solutions x, z, each $<p$.

Remark by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

The following sets of values satisfy the required conditions: When $x=1$, $z=p-2$; when $z=1$, $x=p-2$. When $z=x-2$: $x=\frac{1}{2}+\frac{1}{2}\sqrt{(4p+5)}$, $z=\frac{1}{2}\sqrt{(4p+5)}-\frac{3}{2}$, hence when $4p+5$ is a square there are more than two sets of values x, z, each $<p$.

$\therefore p=11$; $x=1, 4, 9$; $z=9, 2, 1$. $p=41$; $v=1, 7, 39$; $z=39, 5, 1$.
$p=19$; $x=1, 5, 17$; $z=17, 3, 1$. $p=71$; $x=1, 9, 69$; $z=69, 7, 1$.
$p=29$; $x=1, 6, 27$; $z=27, 4, 1$. $p=89$; $x=1, 10, 87$; $z=87, 8, 1$.

AVERAGE AND PROBABILITY.

150. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

If the length of a circular arc be b and the radius vary uniformly, what is the average area of all the segments possible?

Remark by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Evaluating $\triangle=\underset{y=0}{\mathrm{L}}\int_y^{2\pi}\left(1-\frac{\sin y}{y}\right)\frac{dy}{y^3}\Big/\int_y^{2\pi}\frac{dy}{y^2}$, in my solution on p. 41, we find that,

$$\triangle=\underset{y\doteq0}{\mathrm{L}}\frac{\pi yb^2}{2\pi-y}\int_y^{2\pi}\left(1-1+\frac{y^2}{3!}-\frac{y^4}{5!}+\frac{y^6}{7!}-\frac{y^8}{9!}+\text{.........}\right)\frac{dy}{y^3}$$

$$=\underset{y\doteq0}{\mathrm{L}}\frac{\pi yb^2}{2\pi-y}\int_y^{2\pi}\left(\frac{1}{6y}-\frac{y}{5!}+\frac{y^3}{7!}-\frac{y^5}{9!}+\text{.........}\right)dy$$

$$=\underset{y\doteq0}{\mathrm{L}}\frac{\pi yb^2}{2\pi-y}\left[\tfrac{1}{6}\log\frac{2\pi}{y}-\left(\frac{y^2}{2.5!}-\frac{y^4}{4.7!}+\frac{y^6}{6.9!}-\text{.........}\right)_y^{2\pi}\right]$$

$$=\underset{y\doteq0}{\mathrm{L}}\frac{\pi yb^2}{6(2\pi-y)}\log\frac{2\pi}{y}=\underset{y\doteq0}{\mathrm{L}}\tfrac{1}{12}b^2y\log\frac{2\pi}{y}.$$

When $y=0$, $y\log\frac{2\pi}{y}=0\times\infty$. But $y\log\frac{2\pi}{y}=\log\frac{2\pi}{y}\div\frac{1}{y}$; and by differentiating numerator and denominator, $\triangle=\frac{-(2\pi/y^2)/(2\pi/y)}{-(1/y^2)}=y=0$, when $y=0$, instead of $b^2/8\pi$.

Also solved by Henry Heaton, Atlantic, Iowa, with the result $\triangle=0$.

153. Errata. For $1/6a$ read $a/6$.